

# 24RL-1C036

Առաջընթաց՝ արդյունավետ գիտություն 2026  
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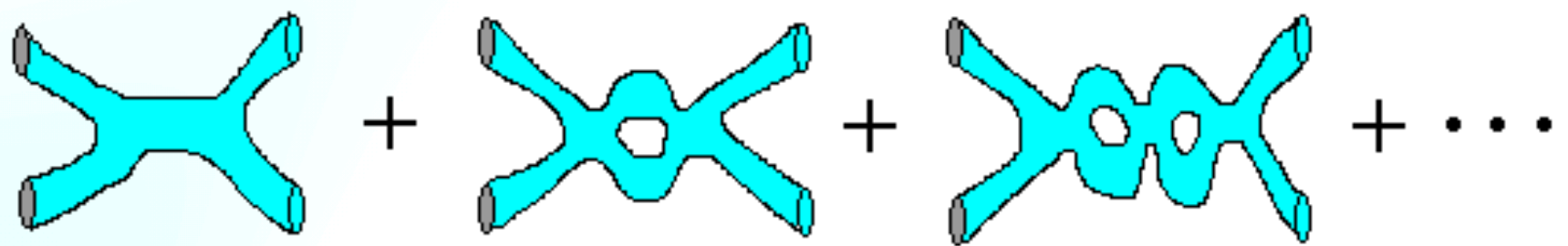
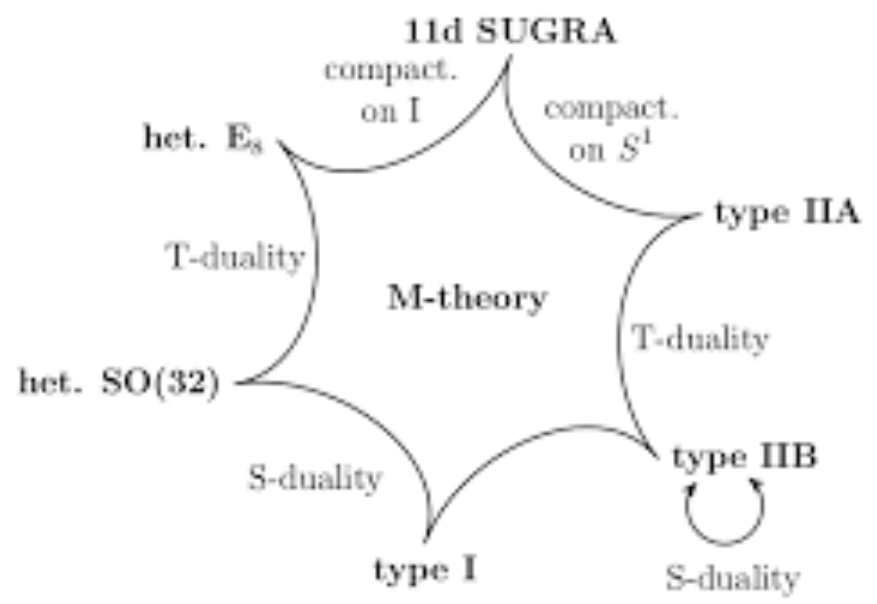
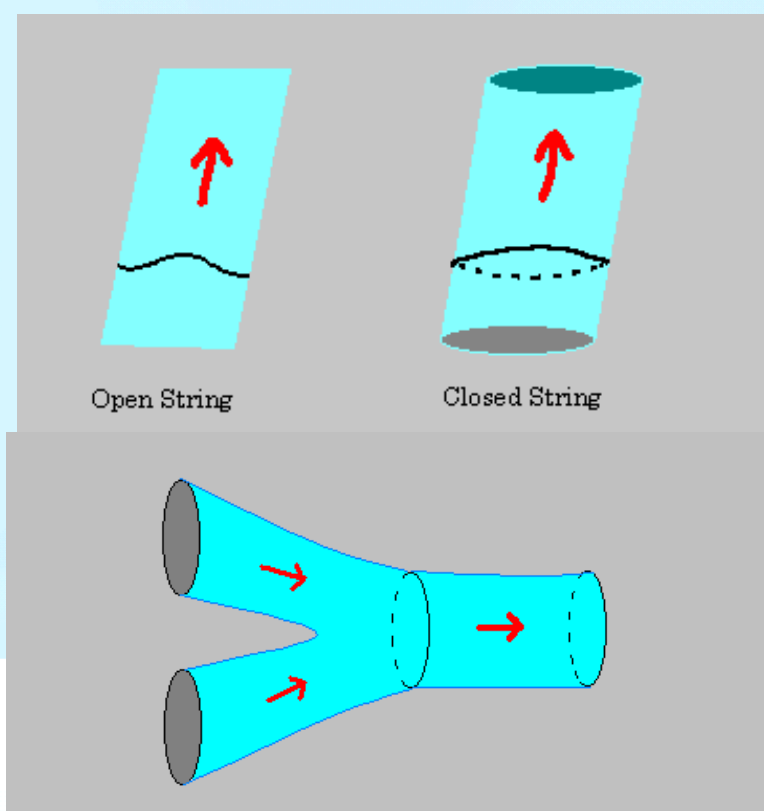
Սամսոնյան Մարինե



# Խմբի անդամները

- Իգնատիոս Անտոնիադիս, պրոֆեսոր, Սորբոնի համալսարան, Ֆրանսիա, Պրինստոն, ԱՄՆ, Չուլալոնգկորն, Թաիլանդ
- Կառլո Անջելանտոնի, պրոֆեսոր, Թուրինի համալսարան, Իտալիա
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# Free Energy (or Partition Function)

In statistical mechanics,  $E_i$  is the energy of state  $i$

$$Z = \sum_i e^{-\beta E_i} \quad \beta = \frac{1}{kT}, \quad k - \text{Boltzman constant}$$

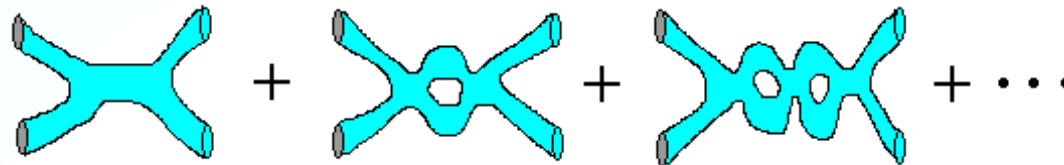
$F = -kT \log Z$  Helmholtz free energy

Euclidean QFT  $Z = \int D\phi e^{-S[\phi]}$  (vacuum-vacuum amplitude)

# Free Energy (or Partition Function)

- Free energy (partition function)  $F = \log Z$  “defines” the theory, more precisely, contains information about the particle content/energy levels, spins, mass... Hence, it depends on the parameters of the theory, at least on the coupling ( $F(\lambda)$  in string theory). For small  $\lambda$

$$F(\lambda) = \sum_{g=0}^{\infty} F_g \lambda^{2g-2} + \mathcal{O}(e^{-\frac{1}{\lambda}})$$



$F_0$ - classical/tree level amplitude/prepotential

$F_1$ - 1-loop correction

$F_g$ -higher genus corrections

# Topological Strings

The theory on the string worldsheet is topological, i.e. physical quantities do not depend on the worldsheet metric.

A-model requires holomorphic maps, i.e.  $X(z)$  only holomorphically

B-model requires quasi constant maps, i. e.  $X \rightarrow z$  is locally constant

Supersymmetric Gauge Theory in  $\Omega \leftrightarrow$  Topological String Theory on CY

Topological strings on CY manifolds  $\epsilon_1 = -\epsilon_2 = \lambda$ , with both parameters  $\epsilon_1, \epsilon_2$  refined parameters of the topological string. They give (count) BPS states more carefully, give also the spin content (explicit calculation later)!

## $\Omega$ background $\epsilon_1, \epsilon_2$

Introduced by Nekrasov, allows to compute the full partition function of

$4D$   $\mathcal{N} = 2$  only vector multiplet

$4D, 5D$   $\mathcal{N} = 2^* = \mathcal{N} = 2 +$  massive adjoint hypermultiplets

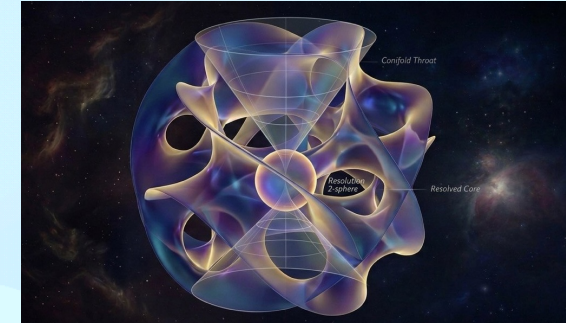
$$(x_1 + ix_2) \rightarrow e^{i\epsilon_1} (x_1 + ix_2) \quad (x_3 + ix_4) \rightarrow e^{i\epsilon_2} (x_3 + ix_4)$$

In  $5D$  going around a circle  $x^5 \rightarrow x^5 + 2\pi R$  simultaneously rotate  $\mathbb{R}^4$

Supersymmetric Gauge Theory in  $\Omega \leftrightarrow$  Topological String Theory on CY  $\leftrightarrow$  M2-M5-brane setup

Topological strings on CY manifolds  $\epsilon_1 = -\epsilon_2 = \lambda$ , with both parameters  $\epsilon_1, \epsilon_2$  refined parameters of the topological string. They give (count) BPS states, give also the spin content (explicit calculation later)!

# Topological String Free Energy



$$F_{\text{GV}}(\lambda, t) = \sum_{\beta > 0} \sum_{g \geq 0} [GV]_{\beta, g} \sum_{n \geq 1} \frac{1}{n \left( 2 \sin \left( \frac{n\lambda}{2} \right) \right)^2} (Q^\beta)^n \left( 2 \sin \left( \frac{n\lambda}{2} \right) \right)^{2g}$$

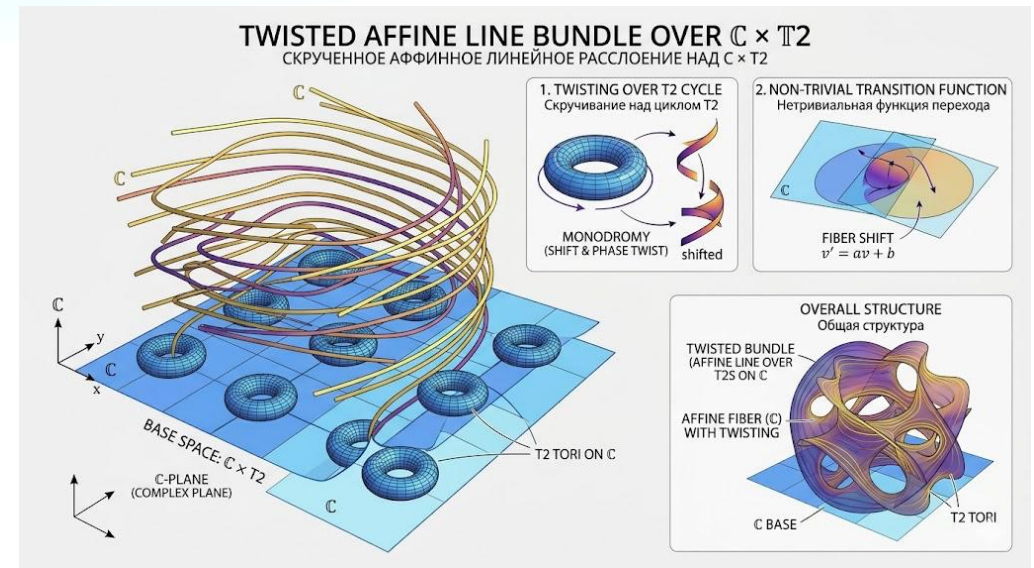
$$F = \sum_{\beta \in H^2(X, \mathbb{Z})} \sum_{n=1}^{\infty} \sum_{J_L} (-1)^{2J_L} \mathcal{N}_\beta^{J_L} e^{-nT_\beta} \frac{q^{-2J_L n} + \dots + q^{2J_L n}}{n(q^{n/2} - q^{-n/2})^2}$$

contribution to F-terms  
from M2-M5 brane setup

$$\sum_{g \geq 0} (-1)^{-g+1} [GV]_{\beta, g} (q^{1/2} - q^{-1/2})^{2g} = (-1)^{2J_L} \mathcal{N}_\beta^{J_L} \sum_{\ell = -J_L}^{J_L} q^{2\ell}$$

$$N = 2^* \quad \text{in} \quad 5D \quad \text{with} \quad U(1)$$

The relevant non-compact CY3-fold can be described as an affine line bundle over  $\mathbb{C} \times T^2$  with a nontrivial twist: the holonomy of the fiber around one 1-cycle of the torus is trivial, while transport around the other 1-cycle acts by a translation of the fiber coordinate by the adjoint mass parameter.



$$F = \log Z = \sum_{n=1}^{\infty} \frac{\mathbf{Q}^n (1 - (T_m T_1)^n) (1 - (T_m T_2)^n)}{n (1 - T_1^n) (1 - T_2^n) (1 - (\mathbf{Q} T_m)^n)}$$

instanton partition function of  
 $5D \quad \mathcal{N} = 2^* \quad U(1)$  .

Poghossian, Samsonyan, 09

$$T_1 = e^{i\epsilon_1 R}, \quad T_2 = e^{i\epsilon_2 R}, \quad T_m = e^{-mR}$$

This gives perturbative, in  $\lambda$  free energy for topological strings compactified on twisted affine line bundle over  $\mathbb{C} \times T^2$  .

In the topological string theory language, it can be written as

$$F = - \sum_{n=1}^{\infty} \frac{e^{-\tilde{t}n}}{n(1 - e^{-\tilde{t}n})} \frac{e^{-mRn} + e^{mRn} - (e^{i\lambda n} + e^{-i\lambda n})}{(e^{i\lambda n/2} - e^{-i\lambda n/2})^2}$$

which can be rewritten as a contour integral around the positive semi-axis without zero

$$F = 2 \oint \frac{ds}{s} \frac{1}{1 - e^{-2\pi is}} \frac{1}{e^{\tilde{t}s} - 1} \frac{\cosh(mRs) - \cos(\lambda s)}{(2 \sin(s\lambda/2))^2} = \oint f(s) ds$$

Double poles at  $\sin$  give the non-perturbative in  $\lambda$  contributions to the free energy!

Calculating the double residue we find a compact formula for the non-perturbative, in  $\lambda$ , correction to the free energy

$$\text{Res} f(s) \Big|_{s=\frac{2\pi k}{\lambda}} = \frac{d}{d\lambda} \left[ \lambda \tilde{F}^0 \left( \frac{2\pi}{\lambda}, \frac{2\pi}{\lambda} (\tilde{t} - i\pi), \frac{mR}{\lambda} \right) \right]$$

with

$$\tilde{F}^0(\lambda, \tilde{t}, u) = \sum_{k=1}^{\infty} \frac{ie^{-k\tilde{t}} \sinh^2(k\pi mR)}{2\pi^2 k^2 \sin(k\pi\lambda) (1 - e^{-k(\tilde{t}+i\pi\lambda)})}$$

$$F_{np,top}(\lambda, \tilde{t}) = -\frac{1}{2\pi i} \partial_\lambda \left( \lambda F^0 \left( \frac{2\pi}{\lambda}, \left( \frac{t^\beta - i\pi}{\lambda} \right) 2\pi \right) \right) \quad \text{Alim 2024}$$

where

$$F^0 = \sum_{\beta} [GV]_{\beta,0} F(\lambda, t^\beta) = \frac{1}{2\pi} \sum_{\beta, n>0} \frac{[GV]_{\beta,0}}{2n^2} \frac{Q^{n\beta}}{\sin(\pi n\lambda)}$$

$Q^\beta = e^{-t^\beta}$  with  $t_\beta$  associated to two dimensional cycles. In our case, we found the following two cycles and the corresponding Kähler parameters

$$\beta = (\ell E + M, \ell E - M, \ell E)$$

$$t_{\ell E+M} = \ell \tilde{t} + mR$$

$$t_{\ell E-M} = \ell \tilde{t} - mR$$

$$t_{\ell E} = \ell \tilde{t}$$

Antoniadis, Samsonyan, 2026

Finding GV invariants with the formula mentioned before.  $\sum_{g \geq 0} (-1)^{-g+1} [GV]_{\beta, g} (q^{1/2} - q^{-1/2})^{2g} = (-1)^{2J_L} \mathcal{N}_{\beta}^{J_L} \sum_{\ell = -J_L}^{J_L} q^{2\ell}$

$$\mathcal{N}_{\ell E+M}^0 = \mathcal{N}_{\ell E-M}^0 = \mathcal{N}_{\ell E}^{1/2} = 1$$

$$\text{For } \mathcal{N}_{\ell E+M}^0 = 1 \quad -[GV]_{\ell E+M, 0} = \mathcal{N}_{\ell E+M}^0 = 1$$

$$\text{For } \mathcal{N}_{\ell E-M}^0 = 1 \quad -[GV]_{\ell E-M, 0} = \mathcal{N}_{\ell E-M}^0 = 1$$

$$\text{For } \mathcal{N}_{\ell E}^{1/2} = 1, \quad -[GV]_{\ell E, 0} + [GV]_{\ell E, 1} (q^{1/2} - q^{-1/2})^2 = -\mathcal{N}_{\ell E}^{1/2} (q + q^{-1}),$$

which gives  $[GV]_{\ell E, 1} = -1$  and  $[GV]_{\ell E, 0} = 2$ . Thus, all genus zero GV invariants that we need are

$$[GV]_{\ell E+M, 0} = -1, \quad [GV]_{\ell E-M, 0} = -1, \quad [GV]_{\ell E, 0} = 2$$

The perturbative partition function for the topological string theory on twisted affine line bundle over  $\mathbb{C} \times T^2$  has the following form

$$F = - \sum_{n,\ell=1}^{\infty} \frac{Q^{n\ell} \tilde{Q}_m^{-n} + Q^{n\ell} \tilde{Q}_m^n - Q^{n\ell} \chi_{J_L=1/2} \left( \sqrt{tq}^n \right)}{(t^{n/2} - t^{-n/2}) (q^{n/2} - q^{-n/2})}$$

$$\chi_j(q) = \frac{q^{2j+1} - q^{-2j-1}}{q - q^{-1}} \quad \text{is the SU(2) character}$$

$$\text{1st term } \chi_{J_L=0} = 1, \quad \chi_{J_R=0} = 1 \quad \mathcal{N}_{\ell E+M,0} = 1$$

$$\text{2nd term } \chi_{J_L=0} = 1, \quad \chi_{J_R=0} = 1 \quad \mathcal{N}_{\ell E-M,0} = 1$$

3rd term

$$\chi_{J_L=1/2} = (tq)^{1/2} + (tq)^{-1/2} = e^{i(\epsilon_1 - \epsilon_2)/2} + e^{-i(\epsilon_1 - \epsilon_2)/2}, \quad \chi_{J_R=0} = 1 \quad \mathcal{N}_{\ell E} = 1$$

Written in the integral form, it appears that perturbative and non-perturbative (instantons, SYM) free energies look very similar

String derivation gives  $\mathcal{F}_{\text{pert.}} = -2 \int_{-\infty}^{+\infty} \frac{dt}{t} \frac{1}{1-e^{-t}} \frac{\cos(mRt) - 1}{\sinh^2(2R\hbar t)}$  , Angelantonj, Antoniadis, Samsosyan, 17

$$F = 2 \oint \frac{ds}{s} \frac{1}{1-e^{-2\pi is}} \frac{1}{e^{\tilde{t}s} - 1} \frac{\cosh(mRs) - \cos(\lambda s)}{(2 \sin(s\lambda/2))^2}$$

from instantons,  
nonperturbative in SYM

Deforming the contour around the positive semi-axis as an integral along the whole imaginary axis  $2\pi is = t$

$\lambda = 8\pi R\hbar$  the two expressions match, up to the  $\tilde{t}$ -dependent factor in containing the instanton counting parameter.

Or formally, take  $e^{\tilde{t}} \rightarrow 0$  corresponds to  $g_{YM}^2 \rightarrow 0$  from the negative direction since

Another way to obtain the perturbative result is by extrapolation of the instanton sum:

expanding  $\frac{1}{e^{\tilde{t}s} - 1} = \sum_{n=1}^{\infty} e^{-\tilde{t}sn}$ , extend  $\sum_{n=0}^{\infty} e^{-\tilde{t}sn}$ , then the  $n = 0$  term corresponds to  $\mathcal{F}_{\text{pert.}}$  upon the contour deformation.

# Refined Topological String Free Energy, in progress

$$F_{\text{n.p.}}(\epsilon_1, \epsilon_2) = -i \sum_{\ell, k=1}^{\infty} \frac{(-1)^k}{k} \left[ \frac{e^{-\tilde{t}\ell \frac{2\pi k}{\epsilon_1}} \cosh\left(\frac{2\pi k m R}{\epsilon_1}\right) - (-1)^k \cos\left(\pi k \frac{\epsilon_2}{\epsilon_1}\right)}{1 - e^{-\frac{4\pi^2 i k}{\epsilon_1}}} \frac{1}{\sin\left(\pi k \frac{\epsilon_2}{\epsilon_1}\right)} + \epsilon_1 \leftrightarrow \epsilon_2 \right]$$

Scalar QED parallel electric and magnetic fields, pair creation probability

$$\text{Im} \mathcal{L} \sim \frac{e^2 E B}{16\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \frac{1}{\sinh\left(\frac{B}{E} n \pi\right)} e^{-\frac{m^2 \pi n}{e E}}$$

A hint to find non-perturbative corrections to (physical) string theory free energy as an analog to Schwinger pair creation of QED?


# Վերջին մեկ տարին



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- Khighatyan, Sh. "Bayesian neural regression and classification on GRRT Images: uncertainties and limitations."  
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“The CosmoVerse White Paper”  
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1) <https://cosmoversetensions.eu/for-the-public/teaching-the-universe/>

### The Big Bang Theory



Explore the journey of the universe from its fiery infancy to its current state. This lesson introduces the concept of the Cosmic Microwave Background (CMB) and how the expansion of the universe affects light waves.

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2) Advanced QFT online lectures in progress now by C. Angelantonj

# Շնորհակալություն ուշադրության համար

