21AG-1C062

Ինտեգրելիություն. նոր կիրառություններ տրամաչափային/լարերի տեսությունից մինչև պինդ մարմնի ֆիզիկա և օպտիկա

Ռուբիկ Պողոսյան	ղեկավար,	ֆ.մ.գ.դ. (2005)
Արմեն Ներսեսյան	համաղեկավար	ֆ.մ.գ.դ. (2005)
Ժիրայր Գևորգյան	կատարող	ֆ.մ.գ.դ. (2002)
Արմեն Պողոսյան	կատարող, ֆ.մ.գ.թ. (2019)	(ղեկավար՝ Ն.Իզմաիլյան)
Վահագն Եղիկյան	կատարող, ֆ.մ.գ.թ. (2010)	(ղեկավար՝ Ա.Ներսեսյան)
Վիգեն Գարեյան	երիտասարդ կատարող,	(ղեկավար՝ Ժ. Գևորգյան)
Հարութ Վարդանյան	երիտասարդ կատարող	(մագիստրոս)

Թեմայի կատարման ընթացքը և ձեռքբերումները

Թեմայի շրջանակում հրապարակվել է ավելի քան 30 հոդված, որոնց ճնշող մեծամասնությունը տպագրված են բարձր վարկանիշային գիտական պարբերականներում: Միջազգային գիտաժողովներում ներկայացվել է ավելի քան 25 զեկուցում: Իրականացվել է նաև 2 թեկնածուական և 1 մագիստրոսական թեզերի պաշտպանություն:

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4. Erik Khastyan, Non-compact complex projective spaces as a phase space of integrable systems: supersymmetric extensions. VII DIAS-TH Int. School “Supersymmetry and integrability” (31.01-04.02.2022, Dubna

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9. RDP School and Workshop on Mathematical Physics, August 19-24, 2023, Yerevan, F.Fucito, F.Morales,R. Poghossian - 2d CFT/gauge theory correspondence in Argyres-Douglas points;

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12. E.Khastyan, Kahler (super)manifolds as phase spaces, JINR-Armenian Workshop “Supersymmetry in Integrable Systems”, 20–22.02.2023, Dubna;

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14. RDP School and Workshop on Mathematical Physics, August 19-24, 2023, Yerevan,
F.Fucito, F.Morales,R. Poghossian - 2d CFT/gauge theory correspondence in Argyres- Douglas points;

15. Zh.Gevorkian, Extended symmetries in geometrical optics, JINR-Armenian Workshop Supersymmetry in Integrable Systems, February 20-22,.2023, Dubna;

16. Zh.Gevorkian, Light Absorption by Weakly Rough Metal Surfaces, International Conference on Laser, Optics and Photonics, March 27-28, Paris,France;

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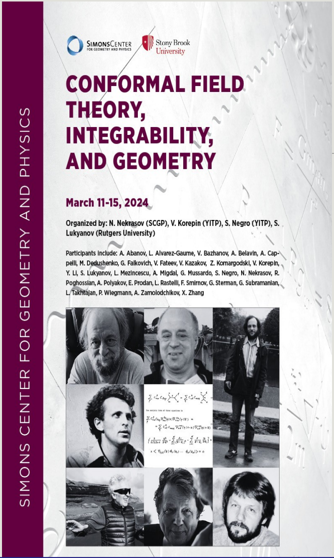
SIMONS CENTER FOR GEOMETRY AND PHYSICS
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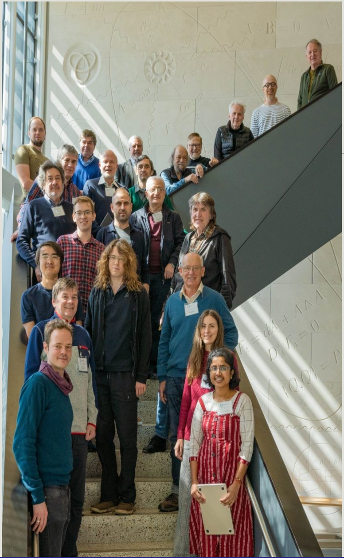
Stony Brook University
CONFORMAL FIELD THEORY, INTEGRABILITY, AND GEOMETRY
March 11-15, 2024
Organized by: N. Nekrasov (SCGP), V. Korepin (YITP), S. Negro (YITP), S. Lukyanov (Rutgers University)
Participants include: A. Abanov, L. Alvarez-Gaume, V. Bazhanov, A. Belavin, A. Cappelli, M. Dedushenko, G. Falkovich, V. Fateev, V. Kazakov, Z. Komargodski, V. Korepin, Y. Li, S. Lukyanov, L. Mezincescu, A. Migdal, G. Mussardo, S. Negro, N. Nekrasov, R. Poghossian, A. Polyakov, E. Prodan, L. Rastelli, F. Smirnov, G. Sterman, G. Subramanian, L. Takhtajan, P. Wiegmann, A. Zamolodchikov, X. Zhang



Թեզերի պաշտպանություն

Թեկնածուական թեզեր են պաշտպանել.

Էրիկ Խաստյանը՝ «Կելերյան փուլային տարածություններով սուպերսիմետրիկ մեխանիկաներ»
19.04.2024, Ղեկավար՝ Արմեն Ներսեսյան

Միեր Դավթյանը՝ «Համաչափ անհամասեռությունների դերը էլեկտրամագնիսական ալիքների
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A new application of 2d CFT/Gauge Theory correspondence

Rubik Poghossian

PES 2026
YerPhI, Armenia

May 13, 2026

Outline

1. Conformal Field Theory (CFT) in 2d
2. Liouville theory, AGT correspondence and irregular conformal blocks
3. BPZ differential equation and recursion relation
4. Application in BH physics

Conformal Field Theory (CFT) in 2d

- A two-dimensional conformal field theory is a quantum field theory that is invariant under local conformal transformations.
- A transformation that locally preserves angles is called conformal
- In 2d any holomorphic transformation preserves angles.

Lets consider a holomorphic transformation $z \to f(z)$ then the metric transforms as

$$ds^2 = dzd\bar{z} \to \left|\frac{\partial f}{\partial z}\right|^2 dzd\bar{z}\,.$$

Let us perform a Laurent expansion of $\epsilon(z)$. Then the infinitesimal conformal transformation can be written as

$$z' = z + \epsilon(z)\,; \qquad \epsilon(z) = \sum_{n\in\mathbb{Z}} c_n z^{n+1}\,,$$

$$\bar{z}' = \bar{z} + \bar{\epsilon}(\bar{z})\,; \qquad \bar{\epsilon}(\bar{z}) = \sum_{n\in\mathbb{Z}} \bar{c}_n \bar{z}^{n+1}\,.$$

The operators that generate this transformations for a particular n are

$$l_n = z^{n+1}\partial_z\,, \qquad \bar{l}_n = \bar{z}^{n+1}\partial_{\bar{z}}\,.$$

These generators obey commutation relations:

$$[l_n, l_m] = (m-n) l_{n+m}\,, \quad [\bar{l}_n, \bar{l}_m] = (m-n) \bar{l}_{n+m}\,, \quad [l_n, \bar{l}_m] = 0\,,$$

Only the subset $\{l_0, l_{\pm 1}\}$ generates conformal transformations that are globally defined on the Riemann sphere $\mathbb{S}^2 = \mathbb{C} \cup \infty$. These generators satisfy the commutation relation:

$$[l_0, l_{-1}] = -l_{-1}\,; \quad [l_0, l_1] = l_1\,; \quad [l_1, l_{-1}] = 2 l_0\,,$$

which is the $sl(2, \mathbb{C})$ algebra.

- l_{-1} and $\bar{l}_{-1}$ are generators of translations (globally $z \to z + \alpha$);
- l_0 and $\bar{l}_0$ are generators of dilatations (globally $z \to \lambda z$);
- l_1 and $\bar{l}_1$ are generators of the special conformal transformations (globally $z \to \frac{z}{1 - \beta z}$).

Fields transforming under the conformal transformation $z \to f(z)$ according to

$$\Phi(z,\bar{z}) \to \left(\frac{\partial f}{\partial z}\right)^{h} \left(\frac{\partial \bar{f}}{\partial \bar{z}}\right)^{\bar{h}} \tilde{\Phi}(f(z),\bar{f}(\bar{z}))\,,$$

are called primary fields with conformal dimension $(h\,,\bar{h})$.
The invariance under $SL(2,\mathbb{C})/\mathbb{Z}_2$ transformations determines two and three- point functions of quasi-primary fields up to some constants.
For the two-point functions one gets

$$\langle \Phi_1(z_1)\Phi_2(z_2)\rangle = \frac{C_{12}\,\delta_{h_1,h_2}}{(z_{12})^{2h}}\,,$$

where $z_{ij} \equiv z_i - z_j$, $h_1 = h_2 \equiv h$ and C_{12} are constants that can be absorbed into normalization of the fields. And for the three-point function the result is

$$\langle \Phi_1(z_1)\Phi_2(z_2)\Phi_3(z_3)\rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3}\, z_{23}^{h_2+h_3-h_1}\, z_{13}^{h_3+h_1-h_2}}\,.$$

In any CFT the energy-momentum tensor has two nonzero components: the holomorphic ant anti-holomorphic fields $T(z)$ and $\bar{T}(\bar{z})$.

$$T(z)T(0) = \frac{c/2}{z^4} + \frac{2T(0)}{z^2} + \frac{T'(0)}{z} + \cdots$$

Laurent series:

$$T(z) = \sum_{n=-\infty}^{\infty} \frac{L_n}{z^{n+2}}$$

Virasoro algebra:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}\,.$$

The OPE of energy-momentum tensor with a primary field is

$$T(z)\Phi(w) = \frac{h}{(z-w)^2}\Phi(w) + \frac{1}{z-w}\partial_w\Phi(w) + \ldots \tag{4.1}$$

The OPE of two primary fields is

The OPE of two primary fields is

$$\Phi_1(z_1)\Phi_2(z_2) = \sum_i C_{12i}(z_1 - z_2)^{\Delta_i - \Delta_1 - \Delta_2}(\Phi_i(z_2) + ...) \quad (4.3)$$

using the OPE of the first two fields in a four-point function of primary fields yields

$$\langle \Phi_1\Phi_2\Phi_3\Phi_4 \rangle = \sum_s C_{12s}C_{s34}\mathcal{F}^s_{\Delta_s}(\{\Delta_i\}, z)\mathcal{F}^s_{\bar{\Delta}_s}(\{\bar{\Delta}_i\}, \bar{z}) \quad (4.4)$$

AGT correspondence and irregular conformal blocks

The correlation function can be written as a linear combination of conformal blocks

$$\langle V_{\alpha_1}(\infty) V_{\alpha_2}(1) V_{\alpha_3}(q) V_{\alpha_4}(0) \rangle$$

The 4d $N = 2$ partition function of $SU(2)$ gauge theories coincides with standard Liouville theory conformal block (AGT).
The limiting procedure which defines Argyres-Douglas theories from SU(2) gauge theories has a simple interpretation as a collision limit in Liouville theory, which produces irregular vertex operators from the collision of several standard vertex operators. Thus the Argyres-Douglas partition function correspond to irregular conformal blocks.

Liouville CFTs

Liouville theory is defined trough the action

$$S = \frac{1}{2\pi i} \int \left(\partial\varphi\bar{\partial}\varphi + \mu\pi e^{2b\varphi} \right) d^2z$$

The central charge of Liouville CFT is

$$c = 1 + 6Q^2, \quad \text{where} \quad Q = (b + 1/b).$$

Primary fields are $V_\alpha = \exp 2\alpha\varphi$ with dimension

$$\Delta_\alpha = \alpha(Q - \alpha).$$

Primary state: $L_n|\Delta\rangle = 0$ for $n > 0$ and $L_0|\Delta\rangle = \Delta|\Delta\rangle$

Recently we (with Hasmik Poghosyan) have explored and developed a recursive approach for deriving regular and irregular CFT blocks and their dual gauge partition functions. In particular we considered rank $\frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$ cases and derived the corresponding second order differential equations (BPZ analogues) satisfied by the irregular block with a level two degenerate field insertion. The solution we represent as a double series expansion.

BPZ differential equation

We start by reminding the BPZ differential equation for five point function

$$F(x,z)=\langle p_0|V_{deg}(z)V_1(1)V_2(x)|p_3\rangle$$

Remind that for a primary state we have

$$L_0|\Delta\rangle=\Delta|\Delta\rangle \quad , \quad L_n|\Delta\rangle=0 \quad \text{for} \quad n>0$$

The level two degenerate field has dimension

$$L_0|\Delta_{deg}\rangle=-\frac{b}{2}\left(Q+\frac{b}{2}\right)|\Delta_{deg}\rangle$$
$$L_n|\Delta_{deg}\rangle=0 \quad \text{for} \quad n>0$$

and

$$\left(L_{-1}^2+b^2L_{-2}\right)V_{deg}(z)=0$$

The BPZ differential equation is

$$\left(\frac{1}{b^2}\frac{\partial^2}{\partial z^2}-\frac{(2z-1)}{z(z-1)}\frac{\partial}{\partial z}+\frac{x(x-1)}{z(z-1)(z-x)}\frac{\partial}{\partial x}+\left(\frac{h_{k_2}}{(z-x)^2}+\frac{h_{p_3}}{z^2}+\frac{h_{k_0}}{(z-1)^2}-\frac{\delta}{z(z-1)}\right)\right)F(x,z)=0$$

From here one can define $G(x,z)=A(x,z)F(x,z)$ and derive

$$\left(\frac{\partial^2}{\partial z^2}-\left(\frac{B_1-B_3+1}{1-z}+\frac{B_2-B_3+1}{x-z}-\frac{B_3}{z}\right)\frac{\partial}{\partial z}+\frac{b^2x(x-1)}{z(z-1)(z-x)}\frac{\partial}{\partial x}+\frac{A_1}{(z-1)z}\right)G(x,z)=0$$

where

$$\begin{aligned} A &= (1-x)^{-\frac{1}{2}(Q-2k_0)(Q-2k_2)}(1-z)^{\frac{1}{2}b(Q-2k_0)} \\ &\times \; z^{\frac{1}{2}b(Q-2p_3)}x^{-\frac{1}{2}(Q-2k_2)(Q-2p_3)}(z-x)^{\frac{1}{2}b(Q-2k_2)} \end{aligned}$$

$$A_1=\frac{1}{4}\left(b^2-2bk_0-2bk_2-2bp_0-2bp_3+2\right)$$
$$\times\left(b^2-2bk_0-2bk_2+2bp_0-2bp_3+2\right)$$
$$B_1=-2bk_0-2bp_3+1\,,\quad B_2=b^2-2bk_2-2bp_3+1\,,\quad B_3=1-2bp_3$$

We know e.g. form instanton counting that the solution should have the form

$$G(x,z)=z^r x^s \sum_{j\geq 0, j+i\geq 0}^{\infty} z^i x^j d_{i,j}$$

The recursion relation

$$C_0(i,j)d_{i,j}+C_1(i)d_{i-1,j}+C_2(i,j)d_{i,j-1}+C_3(i)d_{i+1,j-1}=0\,,$$

where

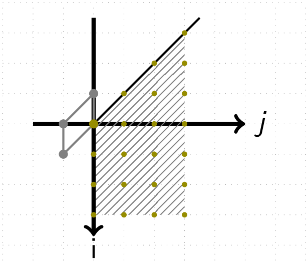
$$\begin{aligned} C_0(i,j) &= -b^2(j+s)-(i+r)(B_2+i+r)\\ C_1(i) &= A_1+(i+r-1)(B_1+B_2-B_3+i+r)\\ C_2(i,j) &= -A_1+b^2(j+s-1)-(i+r)(B_1+i+r)\\ C_3(i) &= (i+r+1)(B_3+i+r)\end{aligned}$$

If we take $i=j=0$ and the condition

$$d_{i,j}=0 \quad \text{if} \quad j<0, \quad \text{or} \quad j+i<0$$

we get that

$$s=-\frac{r\,(B_2+r)}{b^2}$$



$$C_0(i,j)d_{i,j} + C_1(i)d_{i-1,j} + C_2(i,j)d_{i,j-1} + C_3(i)d_{i+1,j-1} = 0\,,$$

with boundary conditions

$$d_{i,j} = 0 \quad \text{if} \quad j < 0, \quad \text{or} \quad j + i < 0$$

we observed

$$d_{-j,j} = \frac{(-r)_j\,(2bp_3 - r)_j}{j!\,(-2\,(r - bk_2 - bp_3))_j}$$

also

$$G(x,z) = z^r x^s \sum_{l,j=0}^{\infty} z^{l-j} x^j d_{l-j,j}$$

We denote by

$$g_j(z) = \sum_{i=-j}^{\infty} d_{i,j} z^i$$

in particular

$$g_0(z) = {}_2F_1\left(a_1, a_2; a_3; z\right)$$

where

$$\begin{aligned} a_1 &= \frac{b^2}{2} - b\,(k_0 + k_2 - p_0 + p_3) + r + 1\,, \\ a_2 &= \frac{b^2}{2} - b\,(k_0 + k_2 + p_0 + p_3) + r + 1\,, \\ a_3 &= b^2 - 2b\,(k_2 + p_3) + 2r + 2 \end{aligned}$$

The following holds

$$d_{l-j,j} = \sum_{m=0}^{j} d_{l-m,0}(c^{(j)}_{m,0} + l c^{(j)}_{m,1})\,, \quad l = 0, 1, \ldots, \quad j = 1, 2, \ldots$$

from here we get

$$g_j(z) = \sum_{m=0}^{j} \left[z^{m-j} c_{m,0}^{(j)} g_0(z) + c_{m,1}^{(j)} z^{1-j} \partial_z (z^m g_0(z)) \right]$$

thus

$$G(x,z) = z^r x^s \sum_{j=0}^{\infty} g_j(z) x^j$$

Remind

$$G(x,z) \sim \langle p_0 | V_1(1) V_{deg}(z) V_2(x) | p_3 \rangle$$

by taking $z \to 0$ we will obtain the four point conformal block.

An intersting application

In NS limit for rank one conformal block we get solution of Confluent Heun Cquation (CHE) in terms of hyper-geometric functions. This results have unexpected application in Black hole physics, since in special case CHE reduces to Teukolski equation, describing metric perturbation in black hole background. This is an ongoing project in collaboration with our Italian colleagues from University of Rome "Tor vergata"

THANKS